Exercise 39

Evaluate the integral.

$$\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{8}{1+x^2}\,dx$$

Solution

Bring the constant in the numerator in front of the integral, write the integrand as a derivative using the rule on page 214, and apply the second part of the fundamental theorem of calculus.

$$\begin{aligned}
\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{8}{1+x^2}\,dx &= 8\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{1}{1+x^2}\,dx \\
&= 8\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{d}{dx}(\tan^{-1}x)\,dx \\
&= 8(\tan^{-1}x)\Big|_{1/\sqrt{3}}^{\sqrt{3}} \\
&= 8\left[\tan^{-1}\left(\sqrt{3}\right) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right] \\
&= 8\left(\frac{\pi}{3} - \frac{\pi}{6}\right) \\
&= \frac{4\pi}{3}
\end{aligned}$$